

# Dynamic Modeling and Torque Estimation of FES-Assisted Arm-Free Standing for Paraplegics

Joon-young Kim, Milos R. Popovic, *Member, IEEE*, and James K. Mills

**Abstract**—This paper presents an application of recent findings in the field of redundant robotic systems' control, toward investigating the feasibility of functional electrical stimulation (FES) assisted arm-free standing for paraplegics. Twelve degrees-of-freedom (DOF) forward and inverse dynamic models of quiet standing have been developed. These models were used to investigate the minimum number of DOF that would need to be actuated in order to generate stable quiet standing in paraplegics despite internal and external disturbances. The results presented herein suggest that the proposed nonlinear dynamic model could achieve guaranteed asymptotic stability with only six active DOF, assuming that the remaining six DOF are passive, i.e., there is no active or passive torques applied to those DOF. The stability analyses were performed using a proportional and derivative (PD) controller coupled with gravity compensation. The results of this analysis suggest that if only six particular DOF are actively controlled in a paraplegic subject, this individual should be able to achieve stable quiet standing despite disturbances. This result has both clinical and system-design implications for the development of a device that will facilitate FES-assisted arm-free quiet standing. The clinical implication is, if a paraplegic patient can exert voluntary control over specified six DOF in the lower limbs, that patient, after intensive physiotherapy, will have the potential to perform quiet standing unassisted. The system-design implication is that FES-assisted arm-free standing for paraplegics is theoretically plausible if one would actively control only six out of 12 DOF in the lower limbs. The proposed solution does not require the locking of joints in the lower limbs (commonly applied in the field) or voluntary control of the upper body to compensate for the internal and external disturbances. Another important finding of this study is the existence of six different combinations of six active DOF able to facilitate stable quiet standing. This dynamic redundancy of the biological bipedal stance allows the selection of an ideal subset of six DOF in designing a neuroprosthesis for standing. This further implies that a considerably less complex FES system than previously anticipated needs to be developed for FES-assisted standing.

**Index Terms**—Bipedal stance, control, functional electrical stimulation (FES), nonlinear dynamics, system redundancy.

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## I. INTRODUCTION

THE APPLICATION of functional electrical stimulation (FES) for the purpose of facilitating quiet standing in individuals suffering from paraplegia is a well researched topic [1]–[8]. These FES systems are also known as neuroprostheses for standing. Although the majority of the FES systems for standing developed thus far use an open-loop control strategy, [3], [6], [9], [10], closed-loop control of paraplegic standing via FES has drawn much attention in recent years. This is primarily because closed-loop control exhibits good disturbance rejection properties and would enable a paraplegic individual, standing with the FES system, to maintain a stable posture despite disturbances. In addition, a closed-loop controlled FES system would allow an individual, who is standing with the help of the device, to use both arms freely to perform activities of daily living during standing, i.e., arm-free standing [1], [2], [4], [5], [8].

Jaeger [1] has suggested that if the body could be approximated as a single degree-of-freedom (DOF<sup>1</sup>) inverted pendulum, then the FES-assisted arm-free standing is at least theoretically possible. This result represents an important milestone in the field of FES-assisted standing that is only recently being tested experimentally [11]. Khang and Zajac [2] used a planar three DOF inverted pendulum model, which included muscle activation and contraction dynamics, to study the optimal control problem in which the energy spent by the muscles is minimized during FES-assisted standing. The outcome of this study was an algorithm which calculated the muscle activation pattern that minimizes energy expenditure for the proposed three DOF model. Matjacic *et al.* [4] have developed a two DOF sagittal plane, dynamic model of quiet standing. They used this model to demonstrate that as long as an appropriate passive stiffness is applied to the ankle joints the model could be stabilized by controlling only the lumbosacral joint, which some paraplegics can control voluntarily. Soetanto *et al.* [8] suggested that a simple proportional and derivative (PD) controller could stabilize the upright posture in the sagittal plane, assuming that the body was modeled as a three DOF inverted pendulum. Mihelj *et al.* [5] extended the work of

<sup>1</sup>In this paper, a joint refers to an articulated point where two rigid body-segments meet, such as at the ankle, knee, or hip joints, whereas a DOF denotes the dimensionality of movement of a joint, e.g., “the ankle joint has two DOF.” Additionally, DOF was also termed to indicate certain degrees of freedom of joints, e.g., “flexion/extension is the only DOF of the knee joint.”

Matjacic *et al.* [4] and demonstrated that by applying optimal control theory, the ankle torque could be minimized.

Before a practical FES system for paraplegic standing can be developed, some limitations in the above-mentioned studies need to be addressed. For example, most of the quiet standing studies describe paraplegic standing as a task performed in the anterior-posterior (sagittal) plane alone. Although a planar model can be used to investigate various relevant phenomena, such as the control strategy of the ankle joint in the sagittal plane during quiet standing [12], it cannot adequately represent the actual dynamics of quiet standing. In particular, it was assumed that during quiet standing both lower limbs generate identical movements and that there is no movement in the frontal plane. Experiments carried in our laboratory, for the purpose of this study, clearly show that this assumption is incorrect. Development of a more realistic dynamic model, although desirable, is a complex task. For example, the standing human body exhibits six-dimensional (6-D) multibody dynamics. The lower limbs consist of a large number of DOF and the orientations of the various joint axes are not constant during limb motion. During limb movements, the muscles move, thus changing the moment of inertia of the body segments. Moreover, during quiet standing, the healthy subjects maintain stable stance without ever moving the feet on the ground. This implies that the body-segmented dynamic system during stable standing is virtually a closed-chain mechanism, i.e., all segments in the lower limbs, such as shanks, thighs, and pelvis are linked together with two feet attached to a rigid surface (the ground). This condition presents a considerable challenge when one attempts to model a closed-chain mechanism of human bipedal quiet standing. Therefore, it is not surprising that the majority of the quiet standing models, available in the literature, avoids addressing the closed-chain mechanism of human bipedal stance and typically assumes that during quiet standing both lower limbs generate identical movements.

Let us for a moment focus on the closed-chain mechanism of quiet stance and discuss what additional complexity it introduces in modeling of kinematics and dynamics of quiet stance. Let us assume that the head-arms-trunk (HAT) can be represented as single rigid body, then in order to regulate its position and orientation in the 6-D space independently an actuator that has six independent DOF is required [13]. However, during quiet stance HAT is actuated by lower limbs that individually comprise of six DOF, i.e., HAT is “actuated” by a 12 DOF dynamic system. During quiet stance, both feet are fully in contact with the ground generating a closed-chain mechanism with a total of 12 DOF. Theoretically, only six independent DOF are required to actuate the HAT. Hence, the discussed dynamic system exhibits redundant dynamics property [14], which implies that no unique set of six DOF torques exists such that the specified kinematic output of the body is generated, i.e., there may be an infinite number of solutions for joint torques that will generate the same kinematic motion of the HAT. In other words, we do not know the minimum number of DOF that has to be actively controlled to facilitate stable quiet standing.

The focus of this paper is on the application of dynamic modeling and control of closed-chain rigid body robotic systems to investigate the minimum dynamic requirements for FES-as-

sisted quiet standing. The modeling and control of the closed-chain mechanism was extensively studied in the robotics field to address an issue of a single objects being manipulated with multiple robotic systems, such as dual arm cooperative manipulators and parallel robots [14]–[19]. In fact, quiet standing and manipulation using dual arm robotic system are two very similar systems because two legs for quiet standing can be regarded as two robot arms that manipulate the HAT cooperatively. Motivated by this similarity, the dynamics and control of such closed-chain robotic systems were examined and the results were applied to FES-assisted quiet standing in paraplegic patients.

Various optimization techniques are commonly applied within the robotics field to address dynamic redundancy. In particular, the problem is posed to find the optimal torques of the joints such that the torques satisfy some condition under which the applied joint torques can be minimized (assuming that all joints are actuated [18], [19]). Instead of applying an optimization technique to address the redundancy problem, we decided to identify the minimum number of DOF that have to be actuated to facilitate stable quiet standing. The issue of minimum number of DOF is even more relevant if one takes into consideration that in paraplegics a number of muscles in the lower limbs are difficult to access or they may be denervated due to spinal cord injury, i.e., not all DOF in FES-assisted quiet standing may be actuated. Recently, Liu *et al.* [16] showed that a nonlinear dual arm manipulation system that has passive joints (not all joints are actuated) could achieve guaranteed asymptotic stability with a PD plus gravity compensation controller. Cheng *et al.* [15] also developed new control schemes for a nonlinear closed-chain mechanism with passive (nonactuated) joints. The results published by Liu *et al.* [16] and Cheng *et al.* [15] have important implications for our study because they suggest a theoretical framework which can be applied to investigate the feasibility of FES-assisted quiet standing. In our study, this theoretical framework was applied to demonstrate that it is theoretically possible to develop a control strategy for FES-assisted standing in 6-D space (no restrictions to planar motions) requiring less than 12 DOF to regulate balance. In addition, a novel method for calculating the inverse dynamics of a rigid-body closed-chain mechanism with passive joints, as proposed by Nakamura *et al.* [17], was applied to determine the minimum number of DOF that is needed for stable FES-assisted standing.

The primary contributions of this study are as follows. First, we developed a nonlinear dynamic model of quiet standing that describes human body motion in 6-D space. Second, through inverse dynamics analysis, we found the minimum number of actuated DOF of the proposed dynamic model such that we could find the unique torque inputs which lead to stable quiet standing. Finally, we demonstrated that these torque inputs can achieve guaranteed asymptotic stability of FES-assisted quiet standing for a complete paraplegic patient in 6-D space. Theoretical stability analysis was performed and simulation results were presented further confirming our theoretical analysis. It is important to mention that this study did not investigate muscle contraction dynamics, muscle response to FES, and the impact of time delays on the system’s performance of the FES-assisted standing. These and other issues have been investigated by our team since



$\mathbf{G}_i(\mathbf{q}_i) := \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ 0^w \tilde{\mathbf{p}}_i & \mathbf{I} \end{bmatrix} \in \mathbb{R}^{6 \times 6}$ ,  $\tilde{\mathbf{A}}$ : matrix product, i.e.,  $\mathbf{A} \times$ ,  
 $\mathbf{q} := \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{bmatrix} \in \mathbb{R}^{12 \times 1}$ ,  $w \tilde{\mathbf{x}}_{\mathbf{O}} \in \mathbb{R}^{6 \times 1}$  is the acceleration term of the HAT,  $w \mathbf{F} \in \mathbb{R}^{12 \times 1}$  is the constraint forces and moments between the legs and the HAT,  $\boldsymbol{\tau} \in \mathbb{R}^{12 \times 1}$  is the joint torque of the legs,  $\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{12 \times 1}$  includes Coriolis-centrifugal and gravity terms of the legs,  $\mathbf{e}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{6 \times 1}$  includes gravity term and gyroscopic moment [24] of the HAT,  $w \mathbf{F}_{\mathbf{O}} \in \mathbb{R}^{6 \times 1}$  is the external force and moment applied to the HAT COM. See [25] for more details about (1).

Equation (1) is the dynamic equation of motion of the system shown in Fig. 1 and it describes the complete kinematic and dynamic characteristics of the model. The limited range of joint motion [26] was implemented in numerical simulation by applying passive joint torques [8]. This prevented the joint from moving beyond typical anatomical joint limits.

### III. INVERSE DYNAMICS

Inverse dynamics analysis can be used to calculate the joint torques required for the proposed model to maintain balance during quiet standing. The fact that the joint variables are not independent, because we are dealing with the closed-chain mechanism, and that the system is dynamically redundant introduces additional challenges with respect to calculation of inverse dynamics. The recursive Newton–Euler algorithm (RNEA) [27] is the most commonly used in the robotics literature to calculate the inverse dynamics for a serial-link manipulator because it is known to be very efficient [28]. However, the RNEA algorithm is much less efficient for closed-chain mechanisms due to the fact that the constraint forces coming from the kinematic constraint must be calculated [28].

The efficient calculation of inverse dynamics is often necessary for real-time control, e.g., computed torque scheme [13], [29]. To overcome this problem, Nakamura *et al.* [17] developed an efficient method to calculate the inverse dynamics for closed-chain mechanisms involving passive joints. This method is based on the Lagrange–D’Alembert principle to formulate inverse dynamic model. The advantage of this method is that it uses dependent coordinates and does not require calculation of the constraint forces. Because of these two attractive features, we adopted Nakamura’s method to calculate the inverse dynamics.

By utilizing Nakamura’s method, the inverse dynamics solution is obtained as

$$\boldsymbol{\tau}_a = \mathbf{W}^T (\mathbf{M}_t \ddot{\mathbf{q}} + \mathbf{C}_t \dot{\mathbf{q}} + \mathbf{N}_t) \in \mathbb{R}^{N_a \times 1} \quad (2)$$

where  $\mathbf{W} := \begin{bmatrix} \mathbf{I} \\ -\mathbf{J}_p^{-1} \mathbf{J}_a \end{bmatrix} \in \mathbb{R}^{12 \times N_a}$ ,  $\mathbf{I} \in \mathbb{R}^{N_a \times N_a}$ : identity matrix,  $\mathbf{q} := [\mathbf{q}_a \ \mathbf{q}_p]^T \in \mathbb{R}^{12 \times 1}$ ,  $\mathbf{q}_a \in \mathbb{R}^{N_a \times 1}$ : active DOF,  $\mathbf{q}_p \in \mathbb{R}^{N_p \times 1}$ : passive DOF (zero torque is applied at the passive DOF),  $N_a$ : number of active DOF,  $N_p$ : number of passive DOF,  $\boldsymbol{\tau}_a \in \mathbb{R}^{N_a \times 1}$ : torques at active DOF,  $\mathbf{J}_a \in \mathbb{R}^{6 \times N_a}$ : Jacobian matrix with respect to the active DOF,  $\mathbf{J}_p \in \mathbb{R}^{6 \times N_p}$ : Jacobian matrix with respect to the passive DOF,  $\mathbf{M}_t(\mathbf{q}) \in \mathbb{R}^{12 \times 12}$ : inertia matrix,  $\mathbf{C}_t(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{12 \times 12}$ : Coriolis-centrifugal force matrix, and  $\mathbf{N}_t(\mathbf{q}) \in \mathbb{R}^{12 \times 1}$ : gravity term.

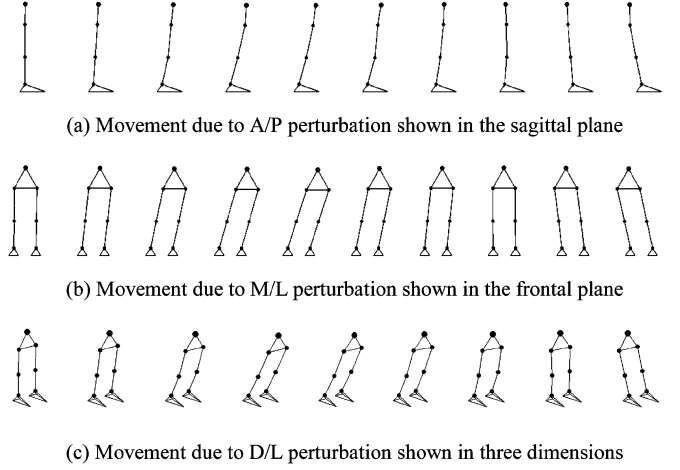


Fig. 2. Animation with stick figures of test  $\mathbf{q}$ . Dots represent ankle, knee and hip joints, and the HAT COM.

Note that  $\dot{\mathbf{M}}_t - 2\mathbf{C}_t$  is skew-symmetric matrix [13]. It should be also noted that the right side of (2) is a function of only  $\mathbf{q}$ ,  $\dot{\mathbf{q}}$ , and  $\ddot{\mathbf{q}}$ . Thus, we need only kinematic and anthropometric information of the system to calculate  $\boldsymbol{\tau}_a$ , i.e., torque at active DOF and the ground reaction forces are not required.

According to (2), when  $N_a = 6$ , there is a unique solution of  $\boldsymbol{\tau}_a$  as long as  $\mathbf{J}_p$  is not singular.  $\mathbf{J}_p$  is referred to as passive Jacobian matrix. The number of active DOF was chosen as six because we were interested in controlling paraplegic standing with the minimum number of DOF. Therefore, before we calculated the inverse dynamics of the (2), we had to find out under what conditions the passive Jacobian matrix is singular.

We investigated which combinations of six active DOF (out of 12 DOF) in the lower limbs could be used to provide a unique torque solution for paraplegic quiet standing (see Fig. 1). This was done by investigating the singularity of the passive Jacobian matrices, i.e., obtaining the rank of the passive Jacobian matrix with respect to the combinations of six active DOF. Since the dynamic model had a total of 12 DOF, there were a total of 924 possible combinations of six active DOF. However, here we assumed that  $q_1, q_6, p_1, p_6$ , which were the DOF corresponding to ankle inversion/eversion and hip rotation about the vertical axis, were passive. This assumption was made because these DOF are rarely actuated by contemporary FES technology. It is also known that the flexor-extensor muscles of ankles, knees, and hips are the dominant muscles used by the central nervous system (CNS) to control balance in the anterior/posterior (A/P) direction while the hip abductor-adductor muscles are used to control balance in the medio/lateral (M/L) direction [30], [31]. Therefore, the number of cases of active DOF combinations to be tested could be reduced to 28 cases.

The investigation of the singularity of the passive Jacobian matrices was performed numerically. Since the passive Jacobian matrix is a function of  $\mathbf{q}$ , we obtained the test  $\mathbf{q}$  by the forward dynamic simulation of the dynamic model shown in Fig. 1 by perturbing the model in three different directions, that is A/P, M/L, and one diagonal (D/L) direction. To prevent the modeled dynamic system from falling over, gravity compensation was applied to the HAT COM during simulation. Typical animation results of these simulations are shown in Fig. 2.

TABLE I  
TOTAL OF SIX POSSIBLE COMBINATIONS OF SIX ACTIVE DOF FOR WHICH PASSIVE JACOBIAN MATRIX HAD A FULL RANK, WERE IDENTIFIED: CASES I TO VI. (D/P: DORSIFLEXION/PLANTARFLEXION, F/E: FLEXION/EXTENSION AND A/A: ABDUCTION/ADDUCTION)

	Ankle D/P		Knee F/E		Hip A/A		Hip F/E	
	Leg	Leg	Leg	Leg	Leg	Leg	Leg	Leg
	A	B	A	B	A	B	A	B
case I		◆	◆	◆		◆	◆	◆
case II	◆		◆	◆		◆	◆	◆
case III	◆	◆		◆		◆	◆	◆
case IV	◆	◆	◆			◆	◆	◆
case V	◆	◆	◆	◆	◆		◆	
case VI	◆	◆	◆	◆		◆	◆	

Table I illustrates six cases of the six active DOF combinations for which the corresponding passive Jacobian matrix obtained a full rank. The results in Table I hold for asymmetric leg configurations, in terms of mass and moment of inertia, because the passive Jacobian matrix is a function of  $\mathbf{q}$ , not of mass or moment of inertia.

The results shown in Table I suggest that there exists a unique and finite set of torques in the lower limbs that generates feasible quiet standing as long as five DOF are used to generate movement in A/P plane and one at the hip that is used to generate movement in M/L plane. Note, ankle D/P, knee F/E, and hip F/E generate motion in the A/P plane, while only the hip A/A generates motion in the M/L plane. This finding suggests that it is theoretically possible to control the quiet standing of a complete paraplegic in 6-D space with only six DOF, which are decoupled into two primary directions, i.e., A/P and M/L directions. Although the results shown in Table I were obtained in simulations, these results were also confirmed by examining the singularity of  $\mathbf{J}_p$  with the joint angles obtained from experiments with healthy subjects [25].

#### IV. FES CONTROL STRATEGY

In this section, we discuss how to control quiet standing using the model shown in Fig. 1. Here, we apply the results obtained in Section III to the control problem of paraplegic standing. We are testing whether the six sets of active six DOF, described in Table I, can be used for quiet standing of paraplegics when the active DOF are assumed to be actuated by an ideal FES system. In this particular case, the ideal FES system is assumed to be a system that elicits muscle contractions such that it generates the exact amount of desired torque at the corresponding DOF without time delay. To learn more about control of FES-assisted standing under conditions of significant time delays and muscle contraction nonlinearities, we recommend reviewing manuscripts [12] and [20]. In these manuscripts, it was shown that a robust PD controller can compensate for time delays up to 185 ms and that the same controller can cope with some muscle nonlinearities such as fatigue and inconsistent response.

In what follows, we propose a PD plus gravity compensation [32]. The controller was used to maintain the upright posture of the paraplegic model in Fig. 1 during quiet standing when six

DOF out of 12 in the lower limbs are actively controlled. Note that the dynamics of the system to be controlled is nonlinear. A nonlinear control problem was considered because perturbation experiments during quiet standing have shown that the joint angles' displacements are not small enough to assume a system's linearity. Also, by addressing a nonlinear control problem and demonstrating that the system is asymptotically stable for a wide range of nonlinear operating points, we can easily guarantee a system's stability for individual linear operating points.

Contemporary motor control theory suggests the central nervous system applies at least two distinctive control strategies to regulate target-tracking tasks. Namely, one control strategy is used to track the desired trajectory of the limb and the other one is used to compensate for gravity. Our recent findings, with respect to balance control during quiet standing, suggest that a PD control may be applied to regulate muscle contractions and target-tracking tasks [12], [20]. In the field of neuroscience, it is well established that the neuronal tracts exist in the spinal cord that provide independent/additional control signals which sole purpose is to compensate for gravity. Therefore, it was a natural choice to use a PD control with a gravity compensation scheme to test if our six combinations of six active DOF, presented in Table I, could generate stable quiet standing. In what follows, it has been shown that a PD control with a gravity compensation scheme can regulate balance during quiet standing.

Equation (2) can be written as

$$\tau_a = \hat{M}_t \ddot{\mathbf{q}}_a + \hat{C}_t \dot{\mathbf{q}}_a + \hat{N}_t \quad (3)$$

where

$$\begin{aligned} \hat{M}_t(\mathbf{q}) &= \mathbf{W}^T \mathbf{M}_t \mathbf{W} \in \mathbb{R}^{6 \times 6} \\ \hat{C}_t(\mathbf{q}, \dot{\mathbf{q}}) &= \mathbf{W}^T \mathbf{M}_t \dot{\mathbf{W}} + \mathbf{W}^T \mathbf{C}_t \mathbf{W} \in \mathbb{R}^{6 \times 6} \\ \hat{N}_t(\mathbf{q}) &= \mathbf{W}^T \mathbf{N}_t \in \mathbb{R}^{6 \times 1}. \end{aligned}$$

It can be shown that  $\hat{M}_t$  is also a symmetric, positive semi-definite matrix and  $\hat{M}_t - 2\hat{C}_t$  is a skew-symmetric matrix [15]. We propose the following control law to apply to the paraplegic dynamic model given by (3)

$$\mathbf{u} = -\mathbf{K}_p \mathbf{e}_a - \mathbf{K}_v \dot{\mathbf{q}}_a + \hat{N}_t \in \mathbb{R}^{6 \times 1} \quad (4)$$

where  $\mathbf{e}_a = \mathbf{q}_a - \mathbf{q}_a^d \in \mathbb{R}^{6 \times 1}$ : error between the  $\mathbf{q}_a$  and the equilibrium point  $\mathbf{q}_a^d$  and  $\mathbf{K}_p, \mathbf{K}_v \in \mathbb{R}^{6 \times 6}$ : gain matrices.

We now introduce a Lyapunov function candidate

$$V = \frac{1}{2} \dot{\mathbf{q}}_a^T \hat{M}_t \dot{\mathbf{q}}_a + \frac{1}{2} \mathbf{e}_a^T \mathbf{K}_p \mathbf{e}_a. \quad (5)$$

Note that although  $\hat{M}_t$  is positive semi-definite,  $V$  is still positive at  $\mathbf{q}_a \neq \mathbf{0}$  because  $(1/2) \dot{\mathbf{q}}_a^T \hat{M}_t \dot{\mathbf{q}}_a \geq 0$  and  $(1/2) \mathbf{e}_a^T \mathbf{K}_p \mathbf{e}_a > 0$ . Differentiating (5) with respect to time, we obtain

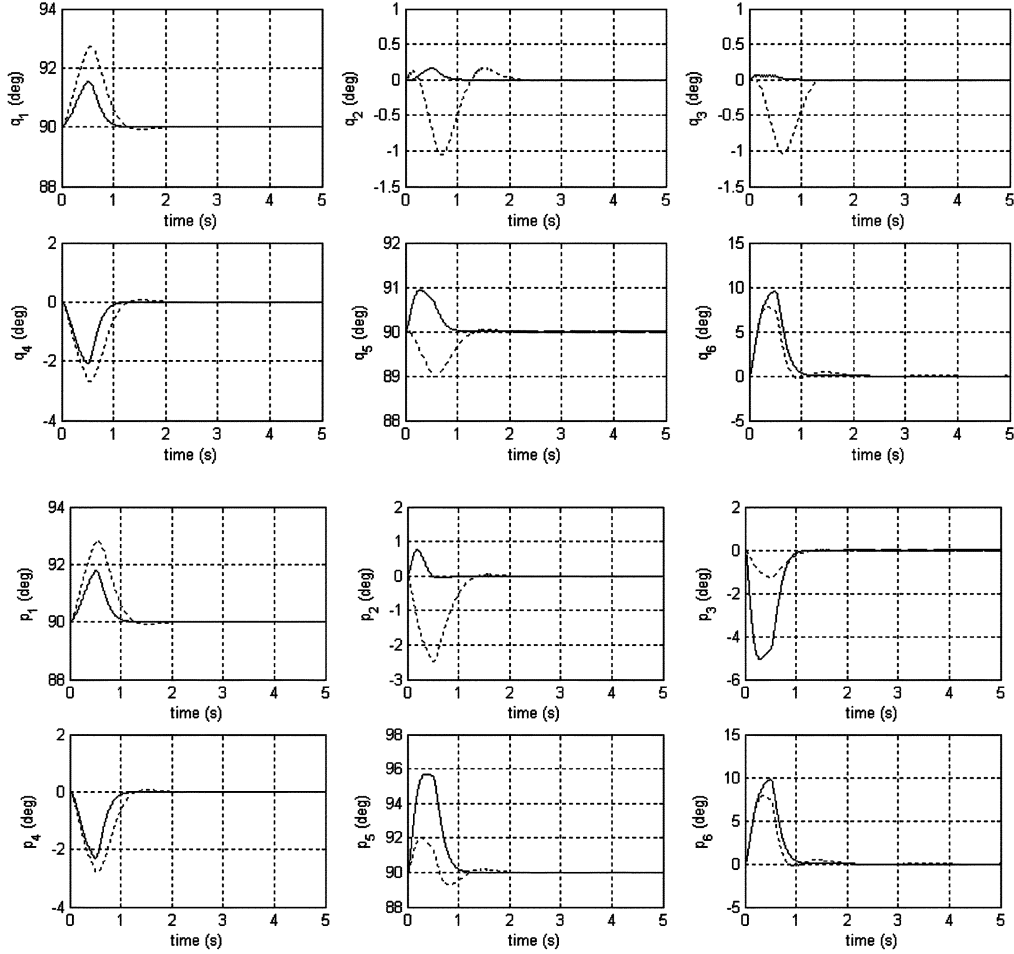
$$\dot{V} = \dot{\mathbf{q}}_a^T \hat{M}_t \dot{\mathbf{q}}_a + \frac{1}{2} \dot{\mathbf{q}}_a^T \dot{\hat{M}}_t \dot{\mathbf{q}}_a + \mathbf{e}_a^T \mathbf{K}_p \dot{\mathbf{e}}_a. \quad (6)$$

Substituting (3) and (4) into (6), we obtain

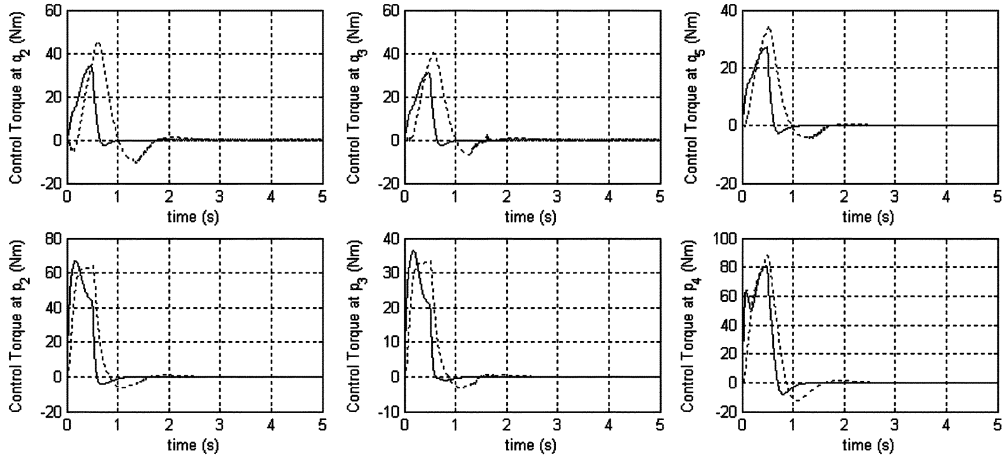
$$\dot{V} = \frac{1}{2} \dot{\mathbf{q}}_a^T \left( \dot{\hat{M}}_t - 2\dot{\hat{C}}_t \right) \dot{\mathbf{q}}_a - \dot{\mathbf{q}}_a^T \mathbf{K}_v \dot{\mathbf{q}}_a. \quad (7)$$

Since  $\dot{\hat{M}}_t - 2\dot{\hat{C}}_t$  is skew-symmetric, we obtain

$$\dot{V} = -\dot{\mathbf{q}}_a^T \mathbf{K}_v \dot{\mathbf{q}}_a \leq 0. \quad (8)$$



(a) Joint variable trajectories



(b) Control Torque at Active DOF

Fig. 3. Simulation results (solid line: computed torque; dashed line: PD plus gravity compensation).

Therefore,  $\dot{V}$  is only negative semi-definite. However,  $\dot{V}$  is actually negative definite because  $\dot{V}$  cannot be zero except at the equilibrium point. In other words, since  $\dot{\mathbf{q}}_a$  is nonzero except at the equilibrium point,  $\dot{\mathbf{q}}_a$  and  $\mathbf{q}_a$  are also nonzero except at the equilibrium point. Thus,  $\dot{V} < 0$  at  $\mathbf{q}_a \neq \mathbf{0}$ . In addition, if  $\mathbf{q}_a = \mathbf{0}$ , then  $\mathbf{q}_p = \mathbf{0}$ . Thus,  $\mathbf{q}_t = \mathbf{0}$ . Therefore, the system with

PD control and gravity compensation is asymptotically stable in the large, i.e., for all states, in the sense of Lyapunov stability.

## V. SIMULATION OF THE CLOSED-LOOP CONTROL

In this section, we performed simulation using the dynamic model in Fig. 1 and the controller discussed in Section IV, i.e.,

PD plus gravity compensation control scheme. The purpose of the simulations was to investigate how well this controller regulates balance and if its responses to disturbances matched the one observed in able-bodied individuals.

The equation of motion (1) was used to represent the dynamics of paraplegic standing, i.e., the system that needs to be controlled. The fixed step-size (0.0002 s) Adams method was used to numerically solve the first order ordinary differential equation (ODE) (1). The fourth order explicit Runge–Kutta method was used to start the simulation [33]. Baumgarte stabilization method [34] was applied to prevent the kinematic constraints from being violated [33]. The paraplegic dynamic model shown in Fig. 1 was assumed to be controlled by both ankle D/P, both knee F/E, and left hip F/E, i.e., case VI in Table I. Due to space limitation, case VI was chosen to present the findings of this phase of the study. The remaining six DOF were assumed to be completely passive, i.e., zero torque was applied to those DOF. The paraplegic subject was assumed to have a mass of 66.7 kg and a height of 1.72 m. The property of each element of the body was obtained based on the anthropometric table [23]. It was assumed that the initial condition of the modeled paraplegic subject was the upright quiet standing, i.e., desired equilibrium state. The equilibrium state of the proposed dynamic model is defined by  $\mathbf{q} = \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{bmatrix} = \mathbf{0}$ , where

$$\mathbf{q}_1 = \mathbf{q}_2 = [90 \ 0 \ 0 \ 0 \ 0 \ 90 \ 0]^T \text{ (deg)}.$$

It was further assumed that the system is perturbed by a sudden external force of 100 N at the HAT COM of the subject in the direction of 45° between  $-x_w$  and  $-z_w$  axis (see Fig. 1). Additionally, the gravity effects resulting from movement of the head, arms, and trunk were also included in the simulation as internal disturbances. These disturbances were assumed to be applied for 0.5 s. Thus, the following internal plus external disturbance forces and moments were used in the simulations,  $w\mathbf{F}_o = [-70.71 \ 0 \ -70.71 \ 10 \ 10 \ 10]^T$  (N, Nm). The total simulation time was 5 s. The error tolerance of the numerical simulation was set to 0.0001. Gain matrices for the PD controller were chosen from [20] and were:  $\mathbf{K}_p = 1000 \mathbf{I}$  (Nm/rad),  $\mathbf{K}_v = 300 \mathbf{I}$  (Nm/rad/s).

The results of simulations are shown in Fig. 3. The joint variables  $\mathbf{q}$  converged to the equilibrium state within 2 s after the model was perturbed. Furthermore, displacements of individual joint variables  $\mathbf{q}$  except for hip rotations about the vertical axis  $\mathbf{q}_6$  and  $\mathbf{p}_6$  did not exceed 5°, and for  $\mathbf{q}_6$  and  $\mathbf{p}_6$  they reached values up to 10°. These joint displacements generate COP displacements which were within low preference zone discussed in [35], i.e., the system was stable and did not lose balance at any instant.

Note that only six DOF were actively controlled by the controller. To model the occurrence of a joint reaching an anatomical limit, passive torques at these joints were generated to model physical joint limits. For instance, the anatomical limit of the knee joint angle was set to  $q_3, p_3 < 0^\circ$ . Thus, these angles were pushed back to zero by the passive torque whenever these joints tried to exceed the anatomical limits. In simulations, few joints reached their anatomical limits except for the knee joints. Since all knee joints' DOF were set as

active DOF in these simulations, none of the passive six DOF reached the anatomical limits. Therefore, the six passive DOF were not controlled actively by the controller or reached their anatomical limits, i.e., zero torques were applied to these joints at all times. The maximum control torque was generated at the hip A/A DOF, i.e., approximately 88 Nm at  $p_4$ .

In ongoing experiments currently being carried on in our laboratory with SCI and able-bodied individuals, we tested whether the obtained maximum torques can be generated using FES. Our preliminary results suggest that the calculated torque maximums in Fig. 3(b) can be achieved without difficulties with the contemporary surface FES technology. For example, in Fig. 3(b), we have shown that the maximum ankle flexion torque of 45 Nm is required to facilitate stable quiet standing. In our experiments, we applied asymmetric biphasic pulses, with pulsewidth 300  $\mu$ s, frequency 35 Hz, and amplitudes in the range from 20 to 48 mA. The stimulations were applied using Compex Motion surface FES system [36] and 10 cm  $\times$  5 cm self-adhesive transcutaneous electrodes. During the experiments, the ankle torques were measured using the Biodex torque dynamometer (Biodex Medical Systems, Shirley, NY). The results obtained suggest, that for the above stimulation parameters, one can generate up to 52 Nm of ankle flexion torque, which is higher than in Fig. 3(b). Please note that in practical FES applications, it is common to use higher stimulation amplitudes and frequencies, and smaller electrodes (5 cm  $\times$  5 cm) than the ones used in these experiments with human subjects. This suggests that even higher ankle flexion torques may be achieved if required.

From the simulation results, the proposed controller demonstrated good perturbation rejection properties. That is, the states converged to equilibrium very quickly after the dynamic model was perturbed by the external forces. This property indicates that the proposed controller is capable of stabilizing the system in the absence of muscle nonlinearities and neuronal delays. Knowing that the PD controller is capable of compensating for significant neuronal delays (up to 185 ms) [20], and since our preliminary experiments indicated that sufficient joint torque could be generated using conventional FES system, we feel confident that at the proposed multi-input–multi-output PD plus gravity compensator has good potential to be used for FES-assisted standing applications.

## VI. CONCLUSION

In this paper, we have addressed the problem of multibody, closed-chain dynamics of quiet standing and have determined the necessary minimum number of DOF that have to be actuated to achieve stable quiet standing, despite internal and external disturbances. We have demonstrated analytically that at least six DOF have to be actively controlled to successfully regulate balance during quiet standing. These six DOF cannot be arbitrarily selected. There exist exactly six sets of active six DOF such that if they are actively controlled one could successfully regulate balance during quiet standing. It is important to mention that five active DOF, out of six, are used to generate movement in A/P plane and the remaining active DOF at the hip is used to generate movement in M/L plane. The remaining six

passive DOF do not have to be actuated at all. Please note that it is not necessary to apply passive stiffness or damping to the passive joints to assist active DOF in maintaining balance. To further validate these results, simulations were performed in which quiet standing was regulated using only six active DOF (suggested in Table I) coupled with a PD plus gravity compensation control scheme. In simulations, it was assumed that the muscle can generate desired torque output and that there is no delay in the muscle response to a command to generate a desired torque. We are aware that these assumptions simplify the problem, however, our previous studies have already addressed these issues successfully using a PD controller [20] and for the purpose of this study, we feel confident that these assumptions were acceptable. The results presented herein have shown that the system can be controlled using the proposed control strategy. Furthermore, the controller generated system's behavior that resembles one observed in able-bodied subjects during quiet standing, and was able to reach the steady state condition as fast as able-bodied subjects do. Therefore, we have concluded that by actuating specific six DOF, one can regulate balance during quiet standing in paraplegics such that it appears to have same properties/behavior as quiet standing observed in able-bodied individuals.

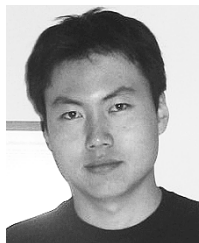
The results presented in this paper have two important scientific implications. First, clinically it suggests that if a paraplegic individual is able to voluntarily control six DOF, and these six DOF belong to one of the six sets described in Table I, it is reasonable to expect that an intensive physiotherapy may help this individual relearn how to perform stable quiet standing. The second implication of these results is that it is not necessary to actuate all DOF in the legs to regulate balance during quiet standing; it is only necessary to control those six DOF described in Table I. This suggests the possible development of a simpler FES system than previously anticipated (a system with less active stimulation channels) to regulate balance during quiet standing. Furthermore, this study suggests that if a particular combination of six DOF cannot be applied to a patient due to denervation of certain muscle groups or to inaccessibility of the muscles for FES application, there exist another five combinations for six DOF that could provide an equally efficient way of performing FES-assisted quiet standing. This feature is critical because many patients have some lower limb muscles denervated (cannot be contracted using contemporary FES technology) due to injury.

Our future work is aimed at combining the results from this study with the findings published in [20] and developing an integrated FES system that would be able to generate arm free quiet standing.

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